Subject Name: **Theory of machines & mechanics**

Subject Code: **ME-3003**

Semester: **3rd**

Unit-IV

Cams: Classification of Cams and Followers, Radial Cam Terminology, Analysis of Follower motion (uniform, modified uniform, simple harmonic, parabolic, cycloidal), Pressure Angle, Radius of Curvature, Cam Profile for radial and offset followers Synthesis of Cam Profile by Graphical Approach, Cams with Specified Contours.

Introduction

A **cam** is a rotating machine element which gives reciprocating or oscillating motion to another element known as a **follower.** The cam and the follower have a line contact and constitute a higher pair. The cams are usually rotated at a uniform speed by a shaft, but the follower motion is predetermined and will be according to the shape of the cam. The cam and follower is one of the simplest as well as one of the most important mechanisms found in modern machinery today. The cams are used to operate the inlet and exhaust valves of internal combustion engines, automatic attachment of machinery, paper cutting machines, spinning and weaving textile machinery, feed mechanism of automatic lathes etc.

Classification of Followers

The followers may be classified as discussed below:

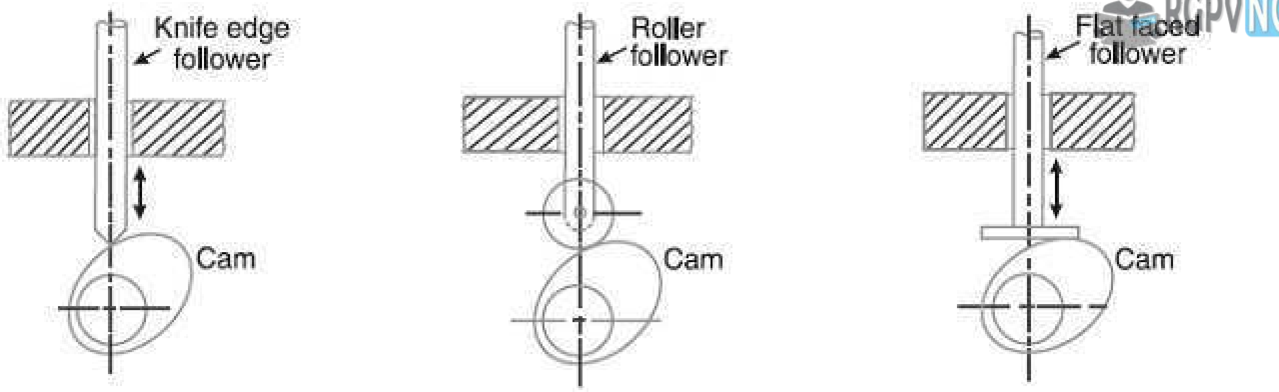
1. According to the surface in contact. The followers, according to the surface in contact, are as follows:

(a) Knife edge follower. Knife edge follower has sharp knife edge at the contact end, as shown in Fig. 4.1 (a). The sliding motion takes place between the contacting surfaces (i.e. the knife edge and the cam surface). It is seldom used in practice because the small area of contacting surface results in excessive wear. A considerable side thrust exists between the follower and the guide in case of these followers.

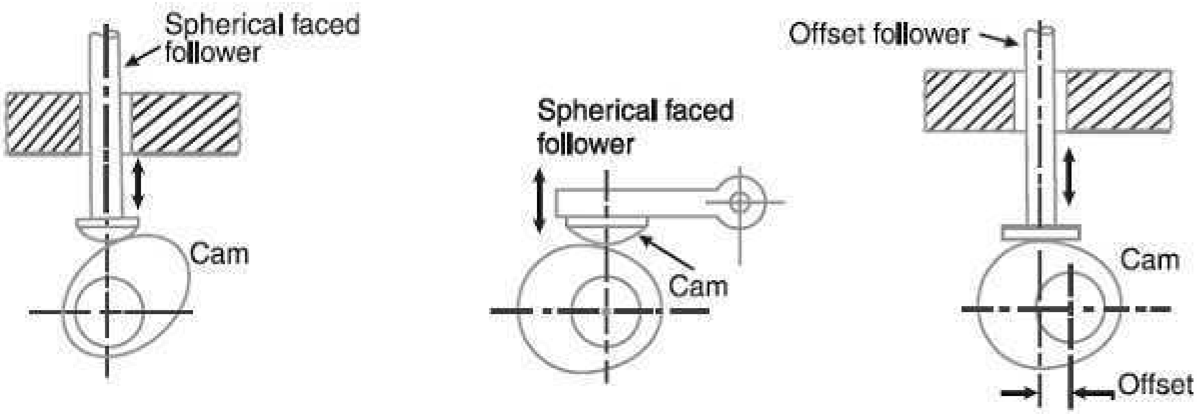
(b) Roller follower. When the contacting end of the follower is a roller, it is called a roller follower, as shown in Fig. 4.1 (b). In case of roller followers, rolling motion takes place between the contacting surfaces (i.e. the roller and the cam), therefore the rate of wear is greatly reduced. In roller followers also the side thrust exists between the follower and the guide. The roller followers are used only where more space is available such as in stationary gas and oil engines and aircraft engines.

(c) Flat faced or mushroom follower. When the contacting end of the follower is a perfectly flat face, it is called a flat-faced follower, as shown in Fig. 4.1 (c). In case of flat-faced followers, the side thrust between the guide and the follower is much reduced. The only side thrust is due to friction between the contact surfaces of the follower and the cam. The relative motion between the surfaces of Cam and follower sliding nature but wear may be reduced by offsetting the axis of the follower, as shown in Fig. 4.1 (f) so that when the cam rotates, the follower also rotates about its own axis. These are generally used where space is limited such as in cams which operate the valves of automobile engines.

(d) Spherical faced follower. When the contacting end of the follower is of spherical shape, it is called a spherical faced follower, as shown in Fig. 4.1 (d). In automobile engines, high surface stresses are produced. In order to minimize these stresses when a flat-faced follower is used, the flat end of the follower is machined to a spherical shape.



(a) Cam with knife edge follower (b) Cam with roller follower (c) Cam with flat faced follower



(d and e) Cam with spherical faced follower (f) Cam with offset follower

Fig. 4.1 Classification of followers

2. According to the motion of the follower. According to the motion follower there are following two types:

(a) Reciprocating or translating follower. When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower. The followers as shown in Fig. 4.1 (a) to 4.1 (d) are all reciprocating or translating followers.

(b) Oscillating or rotating follower. In case of an oscillating follower, the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower. The follower, as shown in Fig. 4.1 (e), is an oscillating or rotating follower.

3. According to the path of motion of the follower. The followers, according to its path of motion, are of the following two types:

(a) Radial follower. When the motion of the follower is along an axis passing through the center of the cam, it is known as a radial follower. The followers, as shown in Fig. 4.1 (a) to 4.1 (e), are all radial followers.

(b) Off-set follower. In off-set followers, the motion of the follower is along an axis away from the axis of the cam center. The follower, as shown in Fig. 4.1 (f), is an off-set follower.

Classification of Cams

Though the cams may be classified in many ways, yet the following two types are important from the subject point of view :

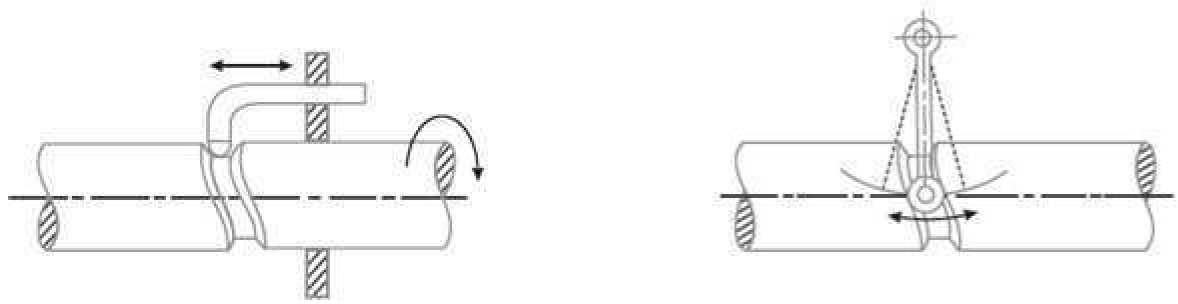


Fig. 4.2 Cylindrical cam

1. Radial or disc cam. The follower reciprocates or oscillates in a direction perpendicular to the cam axis in case of radial cams. The cams as shown in Fig. 4.2 are all radial cams.

2. Cylindrical cam. In cylindrical cams, the follower reciprocates or oscillates in a direction parallel to the cam axis. The follower rides in a groove on its cylindrical surface. A cylindrical grooved cam with a reciprocating and an oscillating follower is shown in Fig. 4.2 (a) and 4.2 (b) respectively.

Terms Used in Radial Cams

Fig. 4.3 shows a radial cam with reciprocating roller follower. The following terms are important in order to draw the cam profile.

1. Base circle: It is the smallest circle that can be drawn to the cam profile.

2. Tracepoint: Tracepoint is used to generate the pitch curve and also it is a reference point on the follower. In case of knife edge follower, the knife edge represents the tracepoint and the pitch curve corresponds to the cam profile. In a roller follower, the center of the roller represents the tracepoint.

3. Pressure Angle: It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.

4. Pitch point: It is a point on the pitch curve having the maximum pressure angle.

5. Pitch circle: It is a circle drawn from the center of the cam through the pitch points.

6. Pitch curve: It is the curve generated by the tracepoint as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas, for a roller follower, they are separated by the radius of the roller.

7. Prime circle: The smallest circle that can be drawn from the center of the cam and tangent to the pitch curve is known as a prime circle. For a knife edge and a flat face follower, the prime circle and the base circle are identical. The prime circle is larger than the base circle of the radius of the roller in case of roller follower.

8. Lift or stroke: It is the maximum travel of the follower from its lowest position to the topmost position.

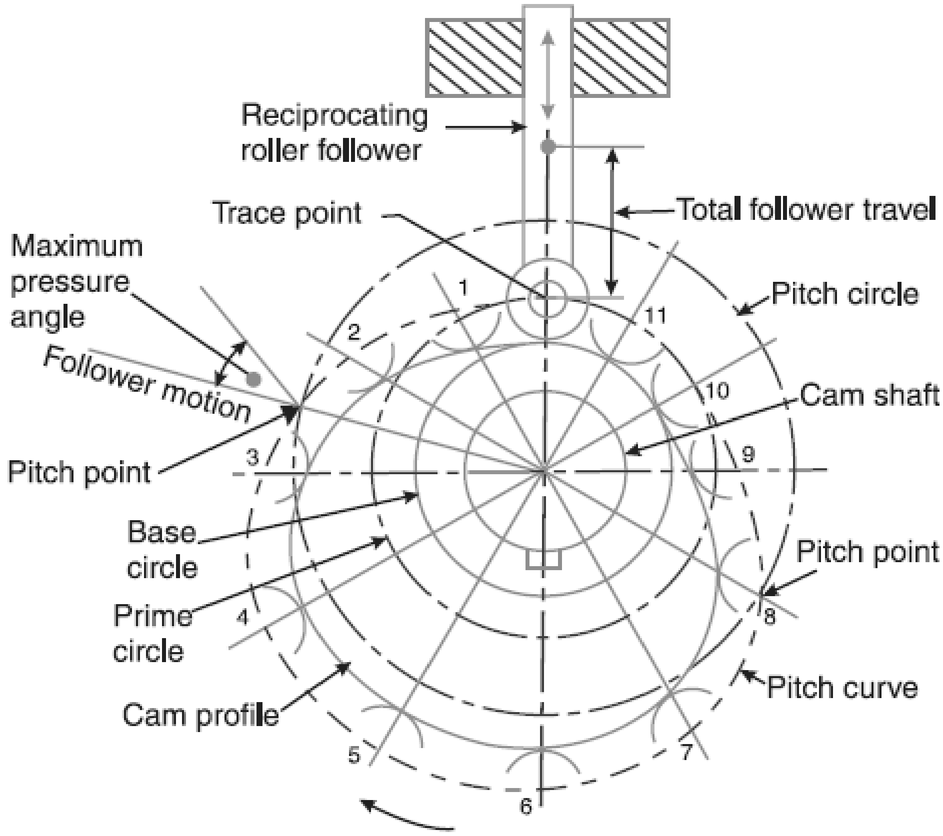


Fig. 4.3 Terms used in radial cams

Motion of the Follower

The follower, during its travel, may have one of the following motions.

1. Uniform velocity,

2. Simple harmonic motion,

3. Uniform acceleration and retardation, and

4. Cycloidal motion

1. Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Velocity:

The displacement, velocity, and acceleration diagrams when a knife-edged follower moves with uniform velocity are shown in Fig. 4.4 (a), (b) and (c) respectively. The time or the angular displacement of the cam is represented on the abscissa (base) in degrees. The ordinate represents the displacement or velocity or acceleration of the follower.

As the follower is moving with uniform velocity during its rise and return stroke, therefore the slope of the displacement curves must be constant. In other words, AB_1 and C_1D must be straight lines. A little consideration will show that the follower remains at rest during part of the cam rotation. Dwell period is the period of time for which the follower remains at rest, as shown by lines B_1C_1 and DE in Fig. 4.4 (a). From Fig. 4.4 (c), we see that the acceleration or retardation of the follower at the beginning and at the end of each stroke is infinite. This occurs due to the fact that the follower is required to start from rest and has to gain a velocity within no time. This is only possible if the acceleration or retardation at the beginning and at the end of each stroke is infinite. These conditions are, however, impracticable.

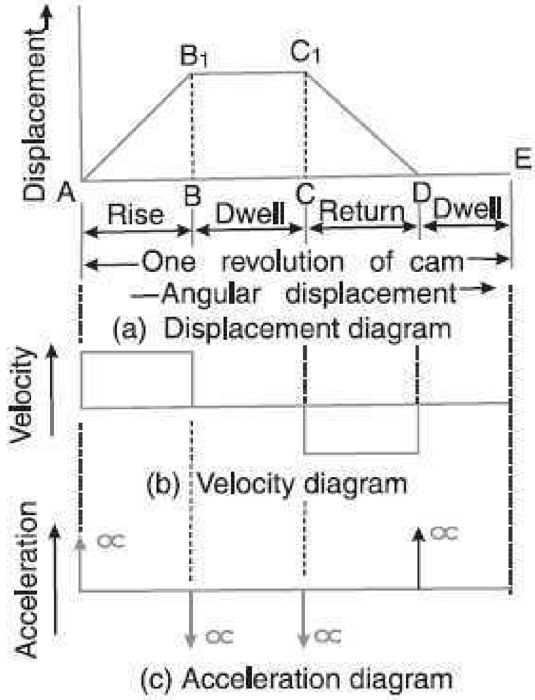


Fig. 4.4 Displacement, velocity and acceleration diagrams when the follower moves with uniform velocity.

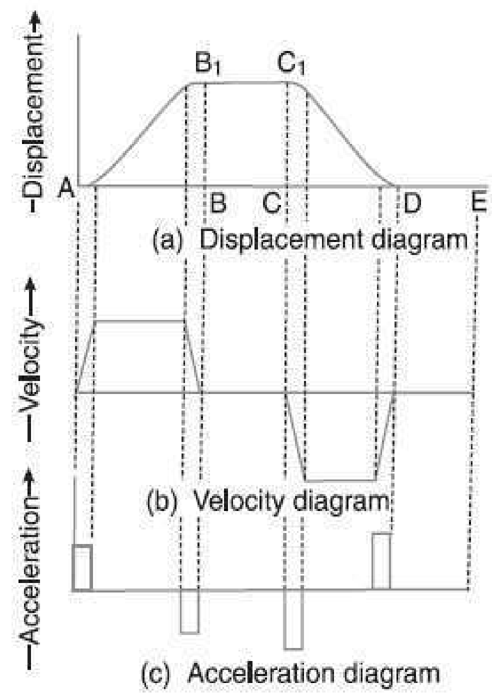


Fig. 4.5 Modified displacement, velocity and acceleration diagrams when the follower moves with uniform velocity.

It is necessary to modify the conditions which govern the motion of the follower for the acceleration and retardation to be in within the limits. This may be done by rounding off the sharp corners of the displacement diagram at the beginning and at the end of each stroke, as shown in Fig. 4.5 (a). By doing this, the velocity of the follower is increasing gradually to its maximum value at the beginning of each stroke and decreases gradually to zero at the end of each stroke as shown in Fig. 4.5 (b). The modified displacement, velocity and acceleration diagrams are shown in Fig. 4.5 (c). The round corners in the displacement diagram are usually parabolic curves because the parabolic motion results in a very low acceleration of the follower for a given stroke and cam speed.

2. Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion

The displacement, velocity, and acceleration diagrams when the follower moves with simple harmonic motion are shown in Fig. 4.6 (a),(b) and (c) respectively. The displacement diagram is drawn as follows:

1. Draw a semi-circle on the follower stroke as diameter.

2. Divide the semi-circle into any number of even equal parts (say eight).

3. Divide the angular displacement of the cam during out stroke and return stroke into the same number of equal parts.

4. The displacement diagram is obtained by projecting the points as shown in Fig. 4.6 (a). The velocity and acceleration diagrams are shown in Fig. 4.6 (b) and 4.6 (c) respectively. The velocity diagram consists of a sine curve and the acceleration diagram is a cosine curve as the follower is moving with simple harmonic motion. We see from Fig. 4.6 (b) that the velocity of the follower is zero at the beginning and at the end of its stroke and increases gradually to a maximum at mid-stroke. On the other hand, the acceleration of the follower is maximum at the beginning and at the end of the stroke and diminishes to zero at mid-stroke.

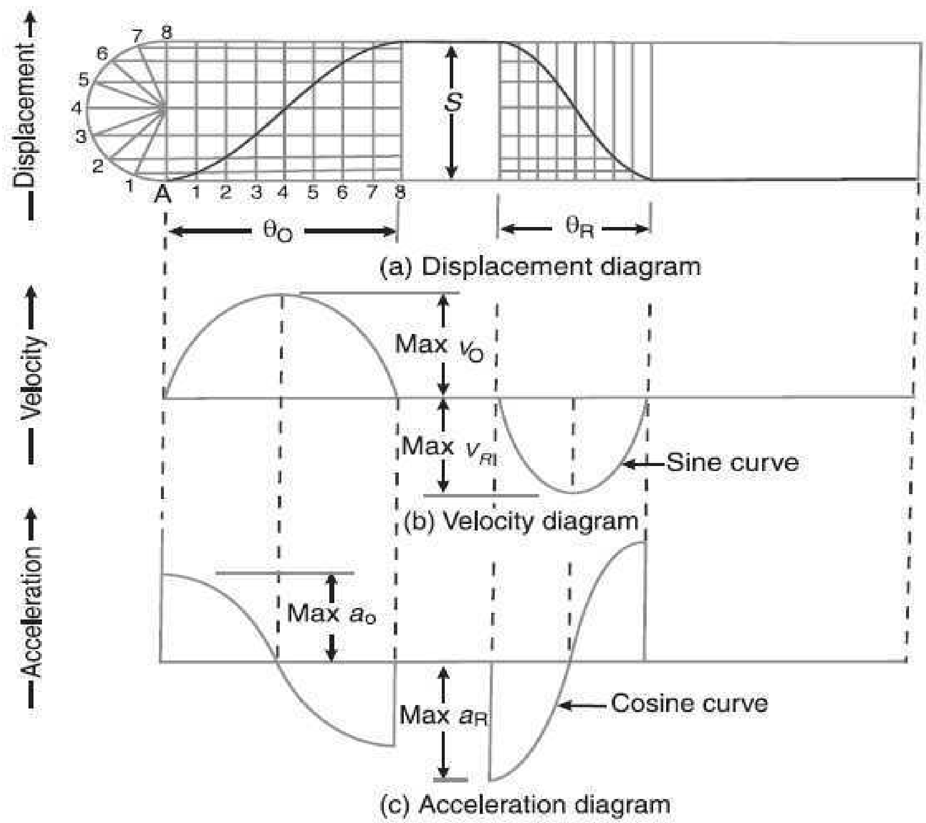


Fig. 4.6 Displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion

Let S = Stroke of the follower,

θ_O and θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively, in radians, and

ω = Angular velocity of the cam in rad/s.

∴Time required for the out stroke of the follower in seconds,

$t_O = \theta_O / \omega$

Consider a point P moving at a uniform speed ω_P radians per sec around the circumference of a circle with the stroke S as diameter, as shown in Fig. 4.6. The point P′ (which is the projection of a point P on the diameter) executes a simple harmonic motion as the point P rotates. The motion of the follower is similar to that of point P′.

∴ Peripheral speed of the point P′

$v_P = \pi S/2 \times 1/t_0 = \pi S/2 \times \omega/\theta_0$

and maximum velocity of the follower on the outstroke,

$v0 = v_P = \pi S/2 \times \omega/\theta_0 = \pi^2\omega^2. S / 2\ \theta_0^2$

We know that the centripetal acceleration of the point P,

$a_P = vP2/OP = (\pi\omega. S / 2\ \theta_0)^2 \times 2/S = \pi^2\omega^2. S / 2\ \theta_0^2$

∴ Maximum acceleration of the follower on the outstroke,

$a_0 = a_P = \pi^2\omega^2. S / 2\ \theta_0^2$

Similarly, maximum velocity of the follower on the return stroke,

$v_R = \pi\omega. S / 2\ \theta_R$

and maximum acceleration of the follower on the return stroke,

$a_R = \pi^2\omega^2. S / 2\ \theta_R^2$

3. Displacement, Velocity and Acceleration Diagrams when the Follower is moving with Uniform Acceleration and Retardation

The displacement, velocity, and acceleration diagrams when the follower moves with uniform acceleration and retardation are shown in Fig. 4.7 (a), (b) and (c) respectively. From the displacement diagram we see that it consists of a parabolic curve and may be drawn as discussed below :

1. Divide the angular displacement of the cam during outstroke (θ_O) into an even number of equal parts (say eight) and draw vertical lines through these points as shown in Fig. 4.7 (a).

2. The stroke of the follower(S) is divided into the equal same number of even parts.

3. Now join Aa which intersects the vertical line through point 1 at B. Similarly, obtain the other points C, D etc. as shown in Fig. 4.7 (a). Now join these points to obtain the parabolic curve for the out stroke of the follower.

4. In a similar way, as discussed above, the displacement diagram for the follower during return stroke may be drawn.

Since the acceleration and retardation are uniforms, therefore the velocity varies directly with the time. The velocity diagram is shown in Fig. 4.7 (b).

Let S = Stroke of the follower,

θ_O and θ_R are the angular displacement of the cam during out stroke and return stroke of the follower respectively, and

ω = Angular velocity of the cam.

We know that time required for the follower during outstroke, $T_O = \theta_O / \omega$

and time required for the follower during the return stroke, $t_R = \theta_R / \omega$

Mean velocity of the follower during outstroke is equal to S/t_O and mean velocity of the follower during return stroke will be equal to S/t_R

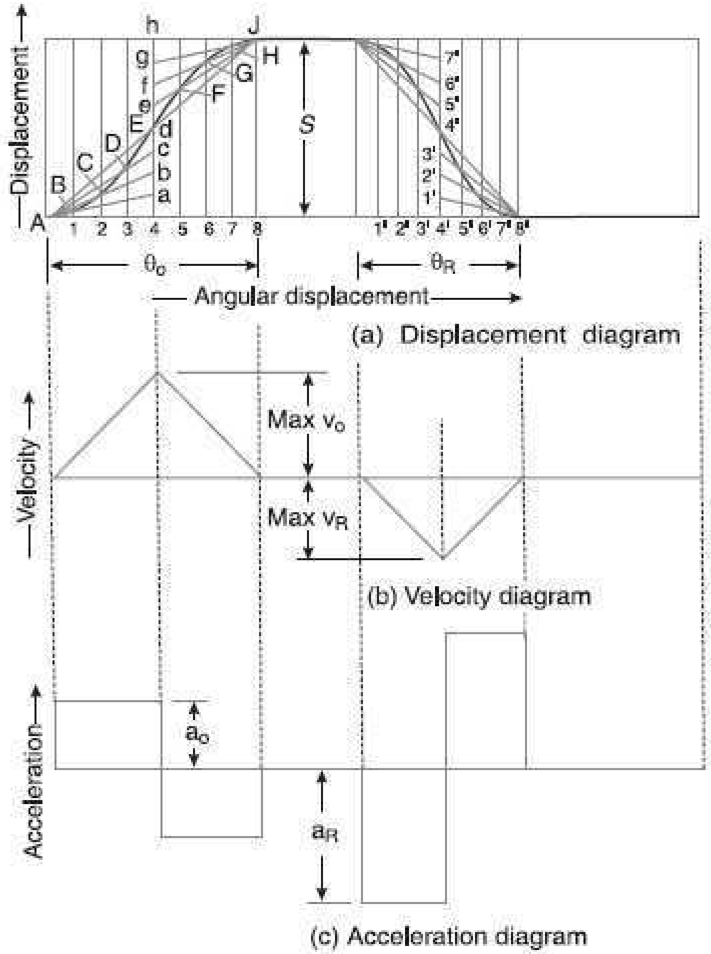


Fig. 4.7 Displacement, velocity and acceleration diagrams when the follower moves with uniform acceleration and retardation

Since the maximum velocity of the follower is equal to twice the mean velocity, therefore the maximum velocity of the follower during outstroke

$v_0 = 2S/t_o = 2\ \omega.\ S / \theta_O$

Similarly, the maximum velocity of the follower during return stroke will be,

$v_R = 2\ \omega.\ S / \theta_R$

We see from the acceleration diagram, as shown in Fig. 4.7 (c), that during the first half of the outstroke there is uniform acceleration and during the second half of the out stroke, there is uniform retardation. Therefore, the maximum velocity of the follower is reached at time $t_O/2$ (during out stroke) and $t_R/2$ (during return stroke).

∴ Maximum acceleration of the follower during outstroke,

$a_O = v_O /(t_O/2) = 2 \times 2\ \omega.\ S / t_O.\ \theta_O = 4\ \omega^2.\ S / \theta_O^2$

Similarly, maximum acceleration of the follower during return stroke,

$a_R = 4\ \omega^2.\ S / \theta_O^2$

4. Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Cycloidal Motion

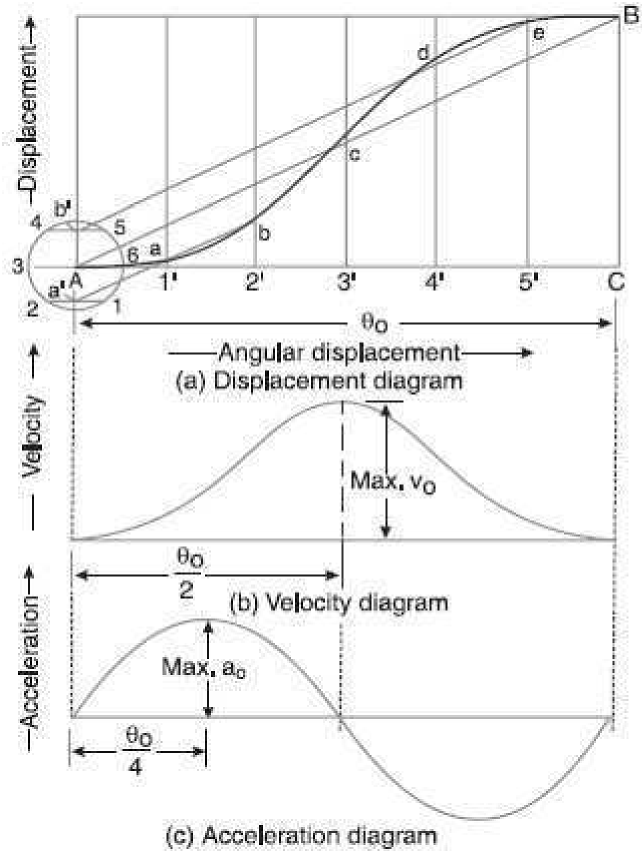


Fig. 4.8 Displacement, velocity and acceleration diagrams when the follower moves with cycloidal motion

When the follower is moving with cycloidal motion the displacement, velocity, and acceleration diagrams are shown in Fig. 4.8 (a), (b) and (c) respectively. We know that cycloid is a curve traced by a point on a circle when the circle rolls without slipping on a straight line.

In case of cams, this straight line is a stroke of the follower which is translating and the circumference of the rolling circle is equal to the stroke (S) of the follower. Therefore the radius of the rolling circle is $S/2\pi$. The displacement diagram is drawn as discussed below:

1. Draw a circle of radius $S/2\pi$ with A as a center.

2. Divide the circle into any number of equal even parts (say six). Project these points horizontally on the vertical center line of the circle. These points are shown by a′ and b′ in Fig. 4.8 (a).

3. Divide the angular displacement of the cam during outstroke into the same number of equal even parts as the circle is divided. Draw vertical lines through these points.

4. Join AB intersecting the vertical line through 3′ at c. From a′ draw a line parallel to AB intersecting the vertical lines through 1′ and 2′ at a and b respectively.

5. Similarly, from b′ draw a line parallel to AB intersecting the vertical lines through 4′ and 5′ at d and e respectively.

6. Join the points A, a, b, c, d, e, and B by a smooth curve. This is the required cycloidal curve for the follower during outstroke.

Let θ = Angle through which the cam rotates in time t seconds, and

ω = Angular velocity of the cam.

We know that displacement of the follower after time t seconds,

$x = S\left[\theta/\theta_O - 1/2\pi \sin(2\pi\theta/\theta_O)\right]$

∴ Velocity of the follower after time t seconds,

$dx/dt = S\left[1/\theta_O \times d\theta/dt - 2\pi/2\pi\theta_O \cos(2\pi\theta/\theta_O)\, d\theta/dt\right]$

$dx/dt = S/\theta_O \times d\theta/dt\left[1 - \cos(2\pi\theta/\theta_O)\right] = \omega S/\theta_O\left[1 - \cos(2\pi\theta/\theta_O)\right]$

The velocity is maximum, when

$\cos(2\pi\,\theta/\theta_O) = -1$ Or $2\pi\,\theta/\theta_O = \pi$ Or $\theta = \theta_O/2$

Substituting $\theta = \theta_O / 2$ in equation, we have maximum velocity of the follower during outstroke,

$v_o = \omega.\, S/\theta_O\,(1 + 1) = 2\omega.\, S/\theta_O$

Similarly, maximum velocity of the follower during return stroke,

$v_R = 2\omega.\, S/\theta_R$

Now, acceleration of the follower after time t sec,

$d^2x/dt^2 = \omega.\, S/\theta_O\,[2\pi/\theta_O \sin(2\pi\,\theta/\theta_O)\, d\theta/dt]$

$d^2x/dt^2 = 2\pi\,\omega^2.\, S/\theta_O^2 \sin(2\pi\,\theta/\theta_O)$

The acceleration is maximum, when

$\sin(2\pi\,\theta/\theta_O) = 1$ Or $2\pi\,\theta/\theta_O = \pi/2$ Or $\theta = \theta_O/4$

Substituting $\theta = \theta_O / 4$ in equation (iii), we have maximum acceleration of the follower during outstroke,

$a_O = 2\pi\,\omega^2.\, S/\theta_O^2$

Similarly, maximum acceleration of the follower during return stroke,

$a_R = 2\pi\,\omega^2.\, S/\theta_R^2$

The velocity and acceleration diagrams are shown in Fig. (b) and (c) respectively.

Construction of Cam Profile for a Radial Cam

To draw the cam profile for a radial cam, the displacement diagram for the given motion of the follower is drawn. Then by constructing the follower in its proper position at each angular position, the profile of the working surface of the cam is drawn.

The principle of kinematic inversion is used in the construction of cam profile, i.e. the cam is imagined to be stationary and the follower is allowed to rotate in the **opposite direction** to the **cam rotation.**

